1 Introduction

1.1 Observational vs. Physical Properties of Stars

What are the key physical properties we can aspire to know about a star? When we look up at the night sky, stars are just little "points of light", but if we look carefully, we can tell that some appear brighter than others, and moreover that some have distinctly different hues or colors than others. Of course, in modern times we now know that stars are really "Suns", with properties that are similar – within some spread – to our own Sun. They only appear much much dimmer because they are much much further away. Indeed they appear as mere "points" because they are so far away that ordinary telescopes almost never can actually resolve a distinct visible surface, the way we can resolve, even with our naked eye, that the Sun has a finite angular size.

Because we can resolve the Sun's surface and see that it is nearly round, it is perhaps not too hard to imagine that it is a real, physical object, albeit a very special one, something we could, in principle "reach out and touch". (Indeed a small amount of solar matter can even travel to the vicinity of the Earth through the solar wind, coronal mass ejections, and energetic particles.) As such, we can more readily imagine trying to assign values of common physical properties – e.g. distance, size, temperature, mass, age, energy emission rate, etc. – that we regularly use to characterize objects here on Earth. Of course, when we actually do so, the values we obtain dwarf anything we have direct experience with, thus stretching our imagination, and challenging the physical intuition and insights we instinctively draw upon to function in our own everyday world. But once we learn to grapple with these huge magnitudes for the Sun, we then have at our disposal that example to provide context and a relative scale to characterize other stars. And eventually as we move on to still larger scales involving stellar clusters or even whole galaxies, which might contain thousands, millions, or indeed billions of individual stars, we can try at each step to develop a relative characterization of the scales involved in these same physical quantities of size, mass, distance, etc.

So let's consider here the properties of stars, identifying first what we can directly *observe* about a given star. Since, as we noted above, most stars are effectively a "point" source without any (easily) detectable angular extent, we might summarize what can be directly observed as three simple properties:

1. **Position on the Sky:** Once corrected for the apparent movement due to the Earth's own motion from rotation and orbiting the Sun, this can be characterized by two coordinates – analogous to latitude and longitude – on a "celestial sphere". Before modern times, measurements of absolute position on the sky had accuracies on order an arcmin; nowadays, it is possible to get down to a few hundreths of an arcsec from ground-based telescopes, and even to about a milli-arcsec (or less in the future) from telescopes in space, where the lack of a distorting atmosphere makes images much sharper. As discussed below, the ability to measure an annual variation in the apparent position of a star due to the Earth's motion around the Sun – a phenomena known as "trignonometric parallax" – provides a key way to infer distance to at least the nearby stars.
2. **Apparent Brightness:** The ancient Greeks introduced a system by which the apparent brightness of stars is categorized in 6 bins called "magnitude", ranging from $m = 1$ for the brightest to $m = 6$ for the dimmest visible to the naked eye. Nowadays we have instruments that can measure a star's brightness quantitatively in terms of the energy per unit area per unit time, a quantity known as the "energy flux" F, with units erg/cm^2/s in CGS or W/m^2 in MKS. Because the eye is adapted to distinguish a large dynamic range of brightness, it turns out its response is *logarithmic*. And since the Greeks decided to give dimmer stars a higher magnitude, we find that magnitude scales with the log of the *inverse* flux, $m \sim \log(1/F) \sim -\log(F)$, with the $\Delta m = 5$ steps between the brightest ($m = 1$) to dimmest ($m = 6$) naked-eye star representing a *factor 100 decrease* in physical flux F. Using long exposures on large telescopes with mirrors several *meters* in diameter, we can nowadays detect individual stars with magnitudes $m > +21$, representing fluxes a million times dimmer than the limiting magnitude $m \approx +6$ visible to the naked eye.
3. **Color or "Spectrum":** Our perception of light in three primary colors comes from the different sensitivity of receptors in our eyes to light in distinct wavelength ranges within the visible spectrum, corresponding to Red, Green, and Blue (RGB). Similarly, in astronomy, the light from a star is often passed through different sets of filters designed to transmit only light within some characteristic band of wavelengths, for example the UBV (Ultraviolet, Blue, Visible) filters that make up the so-called "Johnson photometric system". But much more information can be gained by using a prism or (more commonly) a diffraction grating to split the light into its spectrum, defining the variation in wavelength λ of the flux, F_λ, by measuring its value within narrow wavelength bins of width $\Delta\lambda \ll \lambda$. The "spectral resolution" $\lambda/\Delta\lambda$ available depends on the instrument (spectrometer) as well as the apparent brightness of the light source, but for bright stars with modern spectrometers, the resolution can be 10,000 or more, or indeed, for the Sun, many millions. As discussed below, a key reason for seeking such high spectral resolution is to detect "spectral lines" that arise from the absorption and emission of radiation via transitions between discrete energy levels of the atoms within the star. Such spectral lines

can provide an enormous wealth of information about the composition and physical conditions in the source star.

Indeed, a key theme here is that these 3 apparently rather limited observational properties of point-stars – position, apparent brightness, and color spectrum – can, when combined with a clear understanding of some basic physical principles, allow us to infer many of the key physical properties of stars, for example:

1. **Distance**
2. **Luminosity**
3. **Temperature**
4. **Size** (i.e. Radius)
5. **Elemental Composition** (denoted as X,Y,Z for mass fraction of H, He, and of heavy “metals”)
6. **Velocity** (Both radial (toward/away) and transverse (“proper motion” across the sky)
7. **Mass** (and surface **gravity**)
8. **Age**
9. **Rotation** (Period P and/or equatorial rotation speed V_{rot})
10. **Mass loss properties** (e.g., rate $\dot{M}$ and outflow speed V)
11. **Magnetic field**

These are ranked roughly in order of difficulty for inferring the physical property from one or more of the three types of observational data. It also roughly describes the order in which we will examine them below. In fact, except for perhaps the last two, which we will likely discuss only briefly if at all (though they happen to be two specialities of my own research), a key goal is to provide a basic understanding of the combination of physical theories, observational data, and computational methods that make it possible to infer each of the first 9 physical properties, at least for some stars.

1.2 Scales and Orders of Magnitude

Before proceeding, let us make a brief aside to discuss ways to get our heads around the enormous scales we encounter in astrophysics.

As illustrated in figure 1.1, one approach is to use a geometric progression through *powers of ten*[1], from the scale from our own bodies, which in standard metric (MKS) units is of order 1 meter (m), to the progressively larger scales in our universe.

For example, the meter itself was originally *defined* (in 1793!) as one ten millionth, or 10^{-7}, of the distance from Earth's equator to poles; this thus means

[1] There are many online versions, including a rather dated (1977) but still informative movie titled “Powers of Ten”, which you can readily find by google; for a modern version, see `http://www.htwins.net/scale2/`.

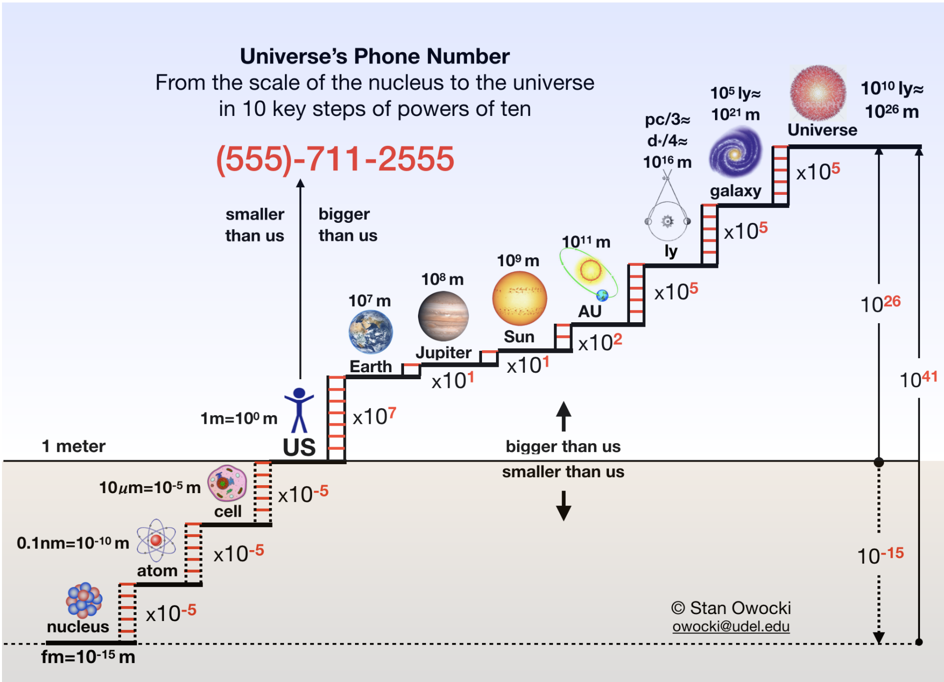

Figure 1.1 Graphic to illustrate key powers-of-ten steps between our own human scale of 1 meter, both upward to the scale of the universe (10^{26} m), and also downward to the scale of an atomic nucleus (10^{-15} m). As a mneumonic, this is cast as a 10-digit "telephone number", with the 3-digit "area code" representing the 3 steps of 10^{-5} from us down to the nucleus, and 7-digit main-number representing 7 key steps to the scale of the universe.

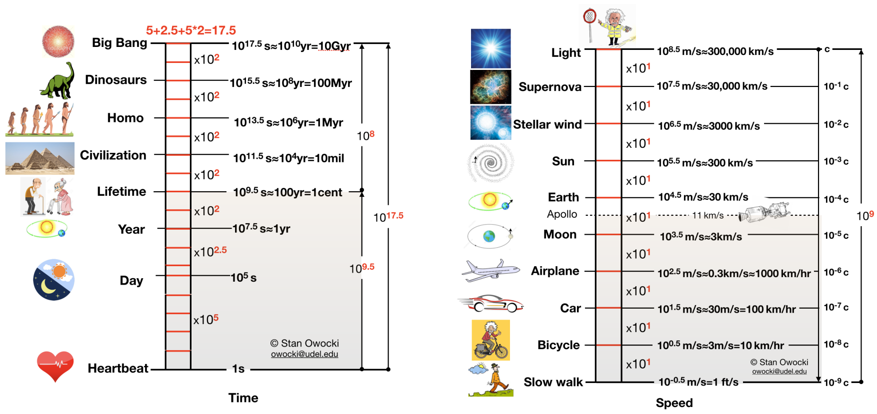

Figure 1.2 Graphics to illustrate the range of scales for time (left) and speed (right).

a total of *seven* steps in powers of ten from the scale of us to that of our Earth.

This is the largest scale for which most of us have direct experience, e.g., from overseas plane travel, or a cross country drive.

The other, rocky inner planets are somewhat smaller but same order as Earth; among the outer, gas giant planets Jupiter is the largest, about a factor ten larger than Earth, while the Sun is about another factor ten larger still, with a diameter $D_\odot \approx 1.4 \times 10^6$ km, about a factor hundred bigger than Earth, or of order 10^9 m.

The Earth-Sun distance, dubbed an "astronomical unit" (AU), is about about a hundred solar diameters, at 150 million km. This is of order $10^8\,\text{km} = 10^{11}\,\text{m}$, or four further powers of ten beyond the scale of our Earth, and so a total of *eleven* orders of magnitude bigger in scale than our own bodies.

An alternative way to characterize this is in terms of the time it takes light, which propagates at a speed $c = 300,000\,\text{km/s}$, to reach us from the Sun; a simple calculation gives $t = d/c = 1.5e8/3e5 = 500\,\text{s}$, which is about eight minutes; so we can say the Sun is 8 *light minutes* from Earth.

By contrast, it takes light from the next nearest star, Proxima Centauri, about *four years* to reach us, meaning it is at a distance of 4 light *years* (ly). A simple calculation shows that one year is $1\ \text{yr} = 365 \times 24 \times 60 \times 60 \approx 3 \times 10^7\,\text{s}$; so multiplying by the speed of light $c = 3 \times 10^5$ km/s gives that $1\ \text{ly} \approx 9 \times 10^{12}$ km, or of order 10^{16} m. Thus the scale between the stars is another five order of magnitude greater than that the Earth-Sun distance, or *sixteen* orders greater than that of ourselves.

The Sun is only one of about 100 billion (10^{11}) stars in our Milky Way galaxy, a disk that is about 1000 ly thick, and about 100,000 ly across. Thus our galaxy is another five orders of magnitude bigger than the scale between individual stars, or about 10^{21} m, thus *twenty-one* orders bigger than us.

The universe itself is about 14 billion years old (14 Gyr), meaning that the most distant galaxies we can see are of order $10^{10}\,\text{ly} \approx 10^{26}\,\text{m}$ away. We thus see that *twenty-six* powers of ten takes us from our own scale to the scale of the entire observable universe!

To recap, powers of ten steps of 7 takes us from us to the Earth; then powers of ten steps 1, 1 and 2 takes us from Earth to the size of Jupiter, Sun, and Earth-Sun distance. Then 3 successive power-ten steps of 5 take us to the distance of the nearest other star; to the size of our galaxy; and finally to the size of the universe. It can be helpful to remember this 711-2555 rule as a mnemonic – like a 7-digit telephone number – to capture the progression between key scales that characterize our place in the universe.

Indeed, we can extend this even to *small* scales, by noting that 5 powers of ten *smaller* takes us successively to the characteristic size of cell, 10^{-5} m=10 micron; then to the size of atoms, $10^{-10}\text{m} = 0.1$ nanometer; and finally to the scale of an atomic nucleus,10 femtometer (a.k.a. "fermi") or 1 fm $= 10^{-15}$ m.

The full sequence of steps over this span thus looks something like a 10-digit phone number with area code: 555-711-2555, representing the power of ten steps from scales of nuclei to atoms to cells to us to Earth to Jupiter to Sun to au (distance to Sun) to light-year ($\sim$ distance between stars) to our Galaxy to the Universe.

Finally, the enormous timescales at play in the universe can likewise be difficult to grasp.

As illustrated in the left panel of figure 1.2, humans experience time in our everyday world on the scale of a second, which is roughly the order of a single heartbeat. We live a maximum of about 100 years, or about 3 billion *seconds*. In comparison, it is estimated that the Earth is about 4.4 billion *years* old, almost as old as the Sun and the rest of the solar system. The Sun is expected to sustain its current energy output for about another 5 billion years, and so have a full lifetime of about 10 billion years. And as discussed below (see §8), the lifetimes of other stars can depend strongly on their mass; the most massive stars (about a hundred solar masses) live only about ten *million* years, while those with mass less than the Sun are expected to last for up to hundred billion years, much much longer than the current age of the universe!

The right panel of figure 1.2 gives a similar graphic for the range of speeds, from our own slow walk, through others (bicycles, cars, airplanes) we experience, then ranging to speeds of the moon, earth and Sun in their orbits, to stellar winds and supernovae, and finally ending with the maximum possible speed, the speed of light, $c = 3 \times 10^8$ m/s. The right axis relates the fraction of the light speed for each of the progression of nine powers from walking to light itself.

The remaining sections below explain how we are able to discover these fundamental properties of stars, beginning with their distance.