4 Inferring Surface Temperature from a Star's Color and/or Spectrum

Let us next consider *why* stars shine with such extreme brightness. Over the long-term (i.e., millions of years), the enormous energy emitted comes from the energy generated (by nuclear fusion) in the stellar core, as discussed further in §18 below. But the more immediate reason stars shine is more direct, namely because their surfaces are so very *hot*. The light they emit is called "thermal radiation", and arises from the jostling of the atoms (and particularly the electrons in and around those atoms) by the violent collisions associated with the star's high temperature[1].

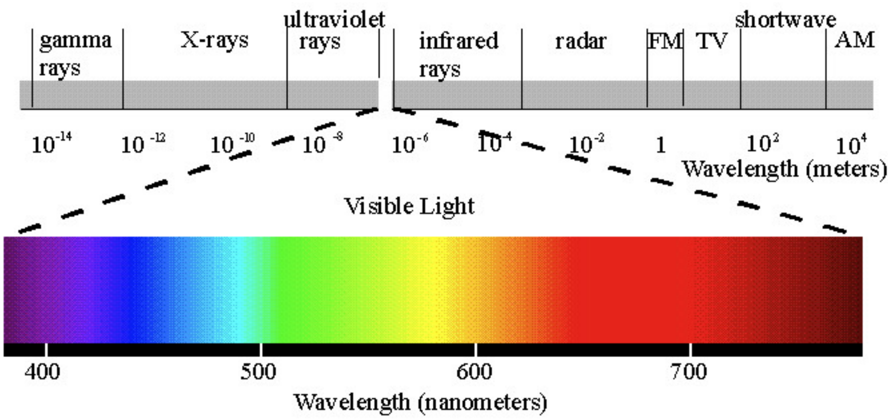

Figure 4.1 The Electromagnetic Spectrum.

[1] In astronomy, temperature is measured in a degree unit called a *Kelvin*, abbreviated K, and defined relative to the centigrade or "Celsius" scale C such that $K = C + 273$. A temperature of $T = 0\,K$ is called "absolute zero", and represents the ideal limit that all thermal motion is completely stopped. To convert from our US use of the Fahrenheit scale F, we first just convert to centigrade using $C = (5/9)(F - 32)$, and then add 273 to get the temperature in K.

4.1 The wave nature of light

To lay the groundwork for a general understanding of the key physical laws governing such thermal radiation and how it depends on temperature, we have to review what is understood about the basic nature of light, and the processes by which it is emitted and absorbed.

The 19th century physicist James Clerk Maxwell developed a set of 4 equations (Maxwell's equations) that showed how variations in Electric and Magnetic fields could lead to oscillating wave solutions, which he indeed indentifed with light, or more generally *Electro-Magnetic (EM) radiation*. The wavelengths λ of these EM waves are key to their properties. As illustrated in figure 4.1, visible light corresponds to wavelengths ranging from $\lambda \approx 400$ nm (violet) to $\lambda \approx 750$ nm (red), but the full spectrum extends much further, including Ultra-Violet (UV), X-rays, and gamma rays at shorter wavelengths, and InfraRed (IR), microwaves, and radio waves at longer wavelengths. White light is made up of a broad mix of visible light ranging from Red through Green to Blue (RGB).

In a vacuum, all these EM waves travel at the *same speed*, namely the speed of light, customarily denoted as c, with a value $c \approx 3 \times 10^5\,\mathrm{km/s} = 3 \times 10^8\,\mathrm{m/s} = 3 \times 10^{10}\,\mathrm{cm/s}$. The wave *period* is the time it takes for a complete wavelength to pass a fixed point at this speed, and so is given by $P = \lambda/c$. We can thus see that the sequence of wave crests passes by at a *frequency* of once per period, $\nu = 1/P$, implying a simple relationship between light's wavelength λ, frequency ν, and speed c,

$$\boxed{\frac{\lambda}{P} = \lambda\nu = c}\,. \tag{4.1}$$

4.2 Light quanta and the Black-Body emission spectrum

The wave nature of light has been confirmed by a wide range experiments. However, at the beginning of the 20th century, work by Einstein, Planck, and others led to the realization that light waves are also *quantized* into discrete wave "bundles" called *photons*. Each photon carries a discrete, indivisible "quantum" of energy that depends on the wave frequency as

$$\boxed{E = h\nu\,,} \tag{4.2}$$

where h is *Planck's constant*, with value $h \approx 6.6 \times 10^{-27}\,\mathrm{erg\,s} = 6.6 \times 10^{-34}$ Joule s.

This quantization of light (and indeed of all energy) has profound and wide-ranging consequences, most notably in the current context for how thermally emitted radiation is distributed in wavelength or frequency. This is known as the "Spectral Energy Distribution" (SED). For a so-called *Black Body* – meaning

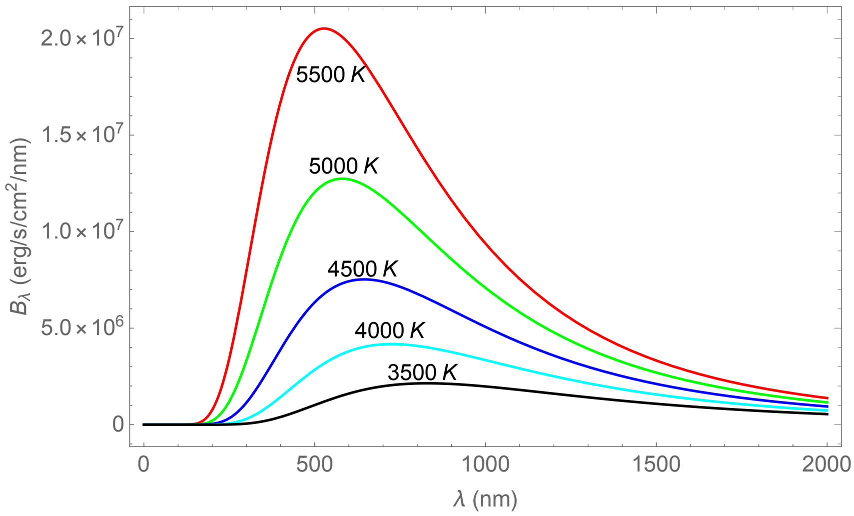

Figure 4.2 The Planck Black-Body Spectral Energy Distribution (SED) vs. wavelength λ, plotted for various temperatures T.

idealized material that is readily able to absorb and emit radiation of all wavelengths –, Planck showed that as thermal motions of the material approach a *Thermodynamic Equilibrium* (TE) in the exchange of energy between radiation and matter, the SED can be described by a function that depends *only* on the gas *temperature* T (and *not*, e.g., on the density, pressure, or chemical composition).

In terms of the wave frequency ν, this *Planck Black-Body* function takes the form

$$\boxed{B_\nu(T) = \frac{2h\nu^3/c^2}{e^{h\nu/kT} - 1}}\,, \tag{4.3}$$

where k is Boltzmann's constant, with value $k = 1.38 \times 10^{-16}$ erg/K $= 1.38 \times 10^{-23}$ Joule/K. For an interval of frequency between ν and $\nu + d\nu$, the quantity $B_\nu d\nu$ gives the emitted energy per unit time per unit area *per unit solid angle*. This means the Planck Black-Body function is fundamentally a measure of *intensity* or *surface brightness*, with B_ν representing the *distribution* of surface brightness over frequency ν, having CGS units erg/cm^2/s/ster/Hz (and MKS units W/m^2/ster/Hz).

Sometimes it is convenient to instead define this Planck distribution in terms of the brightness distribution in a *wavelength* interval between λ and $\lambda + d\lambda$, $B_\lambda d\lambda$. Requiring that this equals $B_\nu d\nu$, and noting that $\nu = c/\lambda$ implies $|d\nu/d\lambda| = c/\lambda^2$, we can use eqn. (4.3) to obtain

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}\,. \tag{4.4}$$

4.3 Inverse-temperature dependence of wavelength for peak flux

Figure 4.2 plots the variation of B_λ vs. wavelength λ for various temperatures T. Note that for higher temperature, the level of B_λ is higher at *all* wavelengths, with greatest increases near the peak level.

Moreover, the location of this peak shifts to *shorter* wavelength with *higher* temperature. We can determine this peak wavelength λ_{max} by solving the equation

$$\left[\frac{dB_\lambda}{d\lambda}\right]_{\lambda=\lambda_{max}} \equiv 0\,. \tag{4.5}$$

Leaving the details as an exercise, the result is

$$\boxed{\lambda_{max} = \frac{2.9\times10^6\,\mathrm{nm\,K}}{T} = \frac{290\,\mathrm{nm}}{T/10,000\,K} \approx \frac{500\,\mathrm{nm}}{T/T_\odot}\,,} \tag{4.6}$$

which is known as *Wien's displacement law.*

For example, the last equality uses the fact that the observed wavelength peak in the Sun's spectrum is $\lambda_{max,\odot} \approx 500\,\mathrm{nm}$, very near the the middle of the visible spectrum.[2] We can solve for a Black-Body-peak estimate for the Sun's surface temperature

$$T_\odot = \frac{2.9\times10^6\,\mathrm{nm\,K}}{500\,\mathrm{nm}} = 5800\,K\,. \tag{4.7}$$

By similarly measuring the peak wavelength λ_{max} in other stars, we can likewise derive an estimate of their surface temperature by

$$\boxed{T = T_\odot\frac{\lambda_{max,\odot}}{\lambda_{max}} \approx 5800\,K\,\frac{500\,\mathrm{nm}}{\lambda_{max}}\,.} \tag{4.8}$$

4.4 Inferring stellar temperatures from photometric colors

In practice, this is not quite the approach to estimating a star's temperature that is most commonly used in astronomy, in part because with real SEDs, it is relatively difficult to identify accurately the peak wavelength. Moreover in surveying a large number of stars, it requires a lot more effort (and telescope time) to measure the full SED, especially for relatively faint stars. A simpler, more common method is just to measure the stellar *color*.

But rather than using the Red, Green, and Blue (RGB) colors we perceive with our eyes, astronomers typically define a set of standard colors that extend to wavebands beyond just the visible spectrum. The most common example is the Johnson 3-color UBV (Ulraviolet, Blue, Visible) system. The left panel of

[2] This is not entirely coincidental, since our eyes evolved to use the wavelengths of light for which the solar illumination is brightest.

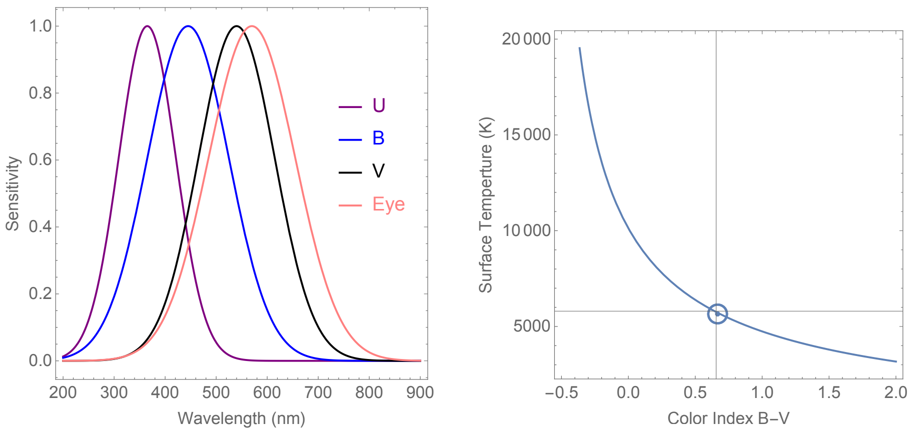

Figure 4.3 *Left:* Comparison of the spectral sensitivity of the human eye with those the UBV filters in the Johnson photometric color system. *Right:* Temperature dependence of the B-V color for a Black-Body emitted spectrum. The circle dot marks the solar values $T_\odot \approx 5800\,\text{K}$ and $(B-V)_\odot \approx 0.656$.

figure 4.3 compares the wavelength sensitivity of such UBV filters to that of the human eye. By passing the star's light through a standard set of filters designed to only let through light for the defined color waveband, the observed apparent brightness in each filter can be used to define a set of color magnitudes, e.g. m_U, m_B, and m_V.

The standard shorthand is simply to denote these color magnitudes just by the capital letter alone, viz. U, B, and V. The *difference* between two color magnitudes, e.g. $B - V \equiv m_B - m_V$, is independent of the stellar distance, but provides a direct diagnostic of the stellar temperature, sometimes called the "color temperature".

Because a larger magnitude corresponds to a lower brightness, stars with a positive B-V actually are less bright in the blue than in the visible, implying a relatively *low* temperature. On the other hand, a negative B-V means blue is brighter, implying a *high* temperature. The right panel of figure 4.3 shows how the temperature of a Black-Body varies with the B-V color of the emitted Black-Body spectrum.

4.5 Questions and Exercises

Quick Question 1: Two photons have wavelength ratio $\lambda_2/\lambda_1 = 2$.
a. What is the ratio of their period P_2/P_1?
b. What is the ratio of their frequency ν_2/ν_1?
c. What is the ratio of their energy E_2/E_1?

Quick Question 2: